

Gauged Vector Models and Higher-Spin Representations in AdS_5

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Abstract: Motivated by the work of Klebanov and Polyakov [hep-th/020114] on the relationship of the large N $O(N)$ vector model in three-dimensions to AdS_4 and higher spin representations, we attempt to find analogous connections for AdS_5 . Since the usual $O(N)$ vector model in four-dimensions is inconsistent, we consider the (consistent) large N gauged vector model and a $\mathcal{N}=1$ supersymmetric analogue in four-dimensions. Both these theories have UV and IR fixed points, and are candidates for a $(\alpha')^{-1}$ expansion in AdS_5 , a conjectured AdS_5/CFT correspondence and higher-spin representations in the bulk theory.

1 Introduction

The most common and productive application of the AdS_d/CFT correspondence is that of a duality between a conformal field theory (CFT) on the boundary of the AdS_d , and a supergravity approximation to a string theory [1]. This is attained in the large N limit, with $\alpha' \ll 1$, which implies that the AdS radius is large compared to the string scale, and is dual to the CFT in a strong coupling regime as well. On the other hand, one might attempt to interpret any CFT as a theory of quantum gravity in an asymptotically AdS space-time, with a radius of curvature of the order of the Planck mass or string scale. In terms of string theory, this suggests an AdS/CFT correspondence to a CFT with an ultraviolet (UV) fixed point, beginning with $\alpha' = \infty$, together with $1/\alpha'$ corrections. This would correspond to a CFT in a weak coupling regime, with the expansion around the UV fixed point of the CFT in contrast to the usual examples of AdS/CFT. More interesting would be examples where the CFT also had an infra-red (IR) fixed point at which the theory was also conformal, perhaps even in the perturbative domain of the CFT. Then one could consider the renormalization group (RG) flow in the $(\alpha')^{-1}$ expansion of the bulk theory from the UV \rightarrow IR as well as in the boundary theory. Such a study might give new insights into the role of string theory in the AdS/CFT correspondence.

In this context Klebanov and Polyakov (KP) [2] discussed an example for AdS_4 ; the large N limit of the $O(N)$ vector model in three-dimensions, which has both a UV and IR fixed point, with the theory being conformal at the IR fixed point. They emphasize the advantage of having a CFT with matter in the fundamental representation of $O(N)$, rather than the adjoint, as it makes available for the bulk theory the work of Vasiliev [3] and others [4, 5] on higher-spin representations.

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This paper is an exploratory project aimed at generalizing in an appropriate way the work of KP to four-dimensions. The four-dimensional, large N , $O(N)$ vector model will not work because it is inconsistent [6], and parenthetically lacks an IR fixed point. A large N gauged vector model coupled to fermions in the fundamental representation does provide a consistent model in four-dimensions [7, 8], as do $\mathcal{N}=1$ supersymmetric analogues. Both classes of models have both UV and IR fixed points, so are candidates for consideration of the kinds of questions raised by KP, but now in four-dimensions. The non-supersymmetric gauged vector model [7, 8] is presented in Sec. 2. However, it is not known whether this theory is conformal at the IR fixed point. An $\mathcal{N}=1$ cousin of this model is presented in Sec. 3, where supersymmetry ensures conformal invariance at the IR fixed point. This theory has Seiberg duality [9], so that there are both electric and magnetic descriptions of the theory. Which one is more useful depends on where one is in the conformal window.

We use the model of Sec. 3 to revisit the issues raised by KP, now in the context of AdS_5/CFT . In Sec. 4 we discuss a possible AdS_5/CFT correspondence for the scalar currents of the theory, and then in Sec. 5 extend this to possible higher-spin representations. At the IR fixed point, the fundamental fields have anomalous dimensions, so that a generalization of the representation theory of Vasiliev [3] and of Sezgin and Sundell [4], though not yet available, is called for as one expects the higher-spin currents to have definite but anomalous dimensions.

Although this work is speculative, it already points to a number of issues worthy of further study.

2 The gauged vector model in $D=4$

The $O(N)$ vector model in four-dimensions has a long history [10, 6, 7]. However, in contrast to the vector model in three-dimensions, the four-dimensional model is unsuitable for study of the issues raised by Klebanov–Polyakov [2] for (at least) two reasons; i) the effective potential has no lowest energy bound as $\phi \rightarrow \infty$, and ii) there is an absence of an infrared (IR) fixed point.

However, both these difficulties are overcome by the *gauged* $U(N)$ vector model [7, 8], coupled to N_f fermions in the fundamental representation. This model can be studied in the large N limit, with N_f/N finite. [That is, the gauged vector model is coupled to the Banks–Zak model [11].] The restrictions of asymptotic freedom, and the reality of the coupling constants throughout the renormalization flows places important restrictions on N_f/N . For massless mesons, these conditions are sufficiently restrictive to imply the existence of an IR fixed point (g_*, λ_*) in both gauge and $\lambda\phi^4$ couplings. This is a consistent massless theory which is scale invariant at the IR fixed point, and is in a non-abelian Coulomb phase. The Lagrangian density of the massless model is [7, 8]

$$\begin{aligned} N^{-1}\mathcal{L} = & \quad |\partial_\mu\phi + igA_\mu\phi|^2 + \frac{1}{2\lambda}\chi^2 \\ & - \chi|\phi|^2 - \frac{1}{4}Tr(F_{\mu\nu}F^{\mu\nu}) + i\sum_{i=1}^{N_f}(\bar{\psi}_i\gamma\cdot D\psi_i) . \end{aligned} \quad (2.1)$$

It is important to note that the coupling constants and fields have been rescaled ($g^2N \rightarrow g^2$; $\lambda N \rightarrow \lambda$, $\phi \rightarrow \sqrt{N}\phi$; $A_\mu \rightarrow \sqrt{N}A_\mu$) so that N is an overall factor of the Lagrangian, and hence N^{-1} is a suitable expansion parameter. Thus in (2.1), g and λ are 't Hooft couplings. The fields ϕ and ψ_i transform in the fundamental representation of $U(N)$, the constraint field χ is a $U(N)$ singlet, and D the covariant derivative. Note the absence of a Yukawa coupling between ϕ and ψ , as both are

in the fundamental representation. Study of the model [7, 8] indicates that there is a zero-mass scalar bound state exactly at the IR fixed point, which appears in $(\phi - \phi)$ scattering. [This state evolves from the tachyon which is present in the 4d vector model with $g=0$ [6, 10].]

The renormalized gauge and scalar ('t Hooft) coupling constants $g(M)$ and $\lambda(M)$ depend on an arbitrary mass-scale M as a result of the renormalization process. They satisfy the RG eq'ns.

$$\beta_g = M \frac{dg}{dM} \quad (2.2a)$$

and

$$\beta_\lambda = M \frac{d\lambda}{dM} \quad (2.2b)$$

where in the large N limit, with N_f/N fixed is [11, 7, 8, 12]

$$\begin{aligned} 16\pi^2\beta_g &= -g^3 \left(\frac{22}{3} - \frac{4}{3} \frac{N_f}{N} \right) \\ &- \frac{4}{3} \frac{g^5}{(4\pi)^2} \left(34 - 13 \frac{N_f}{N} \right) + \dots, \end{aligned} \quad (2.3)$$

and

$$\beta_\lambda = a_0 \lambda^2 - a_1 g^2 \lambda + a_2 g^4, \quad (2.4)$$

where the g^2 dependent coefficients are [7, 8, 12]

$$\begin{cases} (4\pi)^2 a_0 &= 2 + 16 \left(\frac{g}{4\pi} \right)^2 + \dots \\ (4\pi)^2 a_1 &= 12 + \frac{1}{3} \left[256 - 40 \left(\frac{N_f}{N} \right) \right] \left(\frac{g}{4\pi} \right)^2 + \dots \\ (4\pi)^2 a_2 &= 6 + \frac{1}{3} \left[304 - 64 \left(\frac{N_f}{N} \right) \right] \left(\frac{g}{4\pi} \right)^2 + \dots \end{cases} \quad (2.5)$$

Let us review the solution of the coupled RG equations (2.2)–(2.5) [8], as the method will be applicable to the next section. Since $g(M)$ in (2.3) does not depend on λ , one may solve for it first. Define

$$t = \ln M \quad (2.6a)$$

$$x(t) = g^2(M), \quad (2.6b)$$

then from (2.2a)–(2.3)

$$\frac{dx}{dt} = -b_0 x^2 + b_1 x^3 + \dots \quad (2.7a)$$

If

$$\frac{34}{13} < \frac{N_f}{N} < \frac{11}{2}, \quad (2.7b)$$

then $g^2(M)$ is asymptotically free, and there is an IR fixed-point for g^2 , given by [11]

$$\left(\frac{g_*}{4\pi} \right)^2 = \frac{\left(\frac{11}{2} - \frac{N_f}{N} \right)}{13 \left(\frac{N_f}{N} - \frac{34}{13} \right)}. \quad (2.8)$$

To solve for the flow of λ , use the explicit solution for $g^2(M)$, and make the change of variables

$$ds = x(t) dt. \quad (2.9)$$

Then

$$x(s) = \left[A \exp(sb_0) + \frac{b_1}{b_0} \right]^{-1}, \quad (2.10)$$

where

$$A = \left(\frac{1}{x_0} - \frac{b_1}{b_0} \right)$$

with

$$x(s=0) \equiv x_0 = g^2(s=0). \quad (2.11)$$

The variables t and s are related by

$$t = \frac{A}{b_0} [\exp(sb_0) - 1] + \frac{b_1}{b_0} s, \quad (2.12)$$

so that t versus s is single-valued, where $s \rightarrow \pm\infty$ when $t \rightarrow \pm\infty$, with integration constants chosen so that $t = 0$ implies $s = 0$.

Define

$$y(s) = \lambda(s)/g^2(s). \quad (2.13)$$

Then the RG equation for λ becomes

$$\frac{dy}{ds} = a_0[y - y_+(s)][y - y_-(s)] \quad (2.14)$$

where the s -dependent coefficients a_0 , a_1 and a_2 are given by (2.5), b_0 , b_1 by (2.3), (2.7a), and

$$\begin{aligned} y_{\pm}(s) &= \left[\frac{a_1 - b_0 + b_1 x(s)}{2a_0} \right] \\ &\pm \left\{ \left[\frac{a_1 - b_0 + b_1 x(s)}{2a_0} \right]^2 - \frac{a_2}{a_0} \right\}^{1/2} > 0 \end{aligned} \quad (2.15)$$

where reality of the coefficients is required for all values of s . The curve $y_+(s)$ separates the asymptotically free from the non-asymptotic free phase. Flows for $y(s) > y_+(s)$ grow in the UV to $y(s) \xrightarrow{s \rightarrow +\infty} +\infty$, which is not a consistent phase of the model. Flows for $y(s) < y_-(s)$ evolve in the IR to $y(s) \xrightarrow{s \rightarrow -\infty} -\infty$, which is excluded, as negative couplings are not allowed. Therefore $y_-(s) \leq y(s) \leq y_+(s)$ is required for consistency, which describes a non-Abelian Coulomb phase of the theory.

The phase boundary near the UV fixed point is

$$y_+(\infty) = \frac{2}{3} \left(\frac{N_f}{N} - 1 \right) + \left[\frac{4}{9} \left(\frac{N_f}{N} - 1 \right)^2 - 3 \right]^{1/2}, \quad (2.16)$$

which gives the upper-bound of the couplings in the UV,

$$\lambda/g^2 \leq \frac{2}{3} \left(\frac{N_f}{N} - 1 \right) + \left[\frac{4}{9} \left(\frac{N_f}{N} - 1 \right)^2 - 3 \right]^{1/2}. \quad (2.17)$$

Reality of the coupling constants in the UV requires that (2.17) be real, which when combined with (2.7b) gives

$$3.6 \simeq \left(\frac{3\sqrt{3}}{2} + 1 \right) \leq N_f/N \leq \frac{11}{2}, \quad (2.18)$$

which is more restrictive than (2.7a).

The RG flow described by (2.13)-(2.15) in the consistent phase of the theory has a UV and IR fixed point in the large N limit. As an example, the RG flow for $N_f/N = 5$ is shown in Fig. 1, as extracted from ref. [8]. We emphasize that the couplings in the figure are 't Hooft couplings. Note that since N_f/N is close to the upper-bound (2.18), these 't Hooft couplings are small, justifying perturbation theory in the large N limit. The RG flow is already present in the Banks–Zaks model [11], to which we have coupled the gauged vector model. It is not known whether the IR fixed point of our model is stable under $1/N$ corrections.

Notice that because $a_2 \neq 0$ in (2.5) and (2.15), (which comes from purely two or more gauge boson exchange in the scalar scattering), the requirement that couplings be real is a non-trivial constraint on N_f/N , as seen in (2.17). This means that the lower phase-boundary is $y_-(s) \neq 0$, which implies that in the large N limit, the RG flow is disjoint from the case where $\lambda \equiv 0$, which has RG flow along the real axis [$y_-(s) = 0$] of Figure 1. Thus, the g^4 contribution to (2.4) leads to the discontinuous behavior for $\lambda \rightarrow 0$.

An important issue is whether the theory at the IR fixed is conformal, and not just scale invariant. This question is difficult to resolve, and needs additional study. Since we are interested in the possible infinite spin representations for conformal theories, we turn in the next section to a $\mathcal{N} = 1$ supersymmetric “cousin” of the gauged vector model described in this section, where dimensions of chiral operators are protected. If one could show that the theory of this section is conformal at the IR fixed point, one could analyze it in analogy to Secs. 4 and 5.

3 Supersymmetric gauged vector model

Consider $\mathcal{N} = 1$ supersymmetric QCD with gauge group $SU(N)$, N_f flavors of quarks Q^i in the fundamental representation, $\tilde{Q}_{\tilde{i}}$ in the anti-fundamental representation ($i, \tilde{i} = 1$ to N_f), and in addition a massless chiral superfield σ which is a color and flavor singlet. The chiral superfields interact by means of the superpotential

$$W = \sqrt{\frac{\lambda}{N}} \sigma \sum_{i=1}^{N_f} Q^i \tilde{Q}_{\tilde{i}}. \quad (3.1)$$

The coupling in (3.1) has been scaled so that it is the 't Hooft coupling, with the same convention as Sec. 2. Thus the chiral superfield σ plays a role analogous to χ in (2.1), except that here we keep it as a propagating degree of freedom. [If σ were to be integrated out, one would have a $\mathcal{N} = 1$ theory analogous to that studied in Sec. 2]. This model with $\lambda = 0$ was studied extensively by Seiberg [9], while for $\lambda \neq 0$ one encounters some issues reminiscent of those considered by Leigh and Strassler [13]. In our discussion we only consider the non-Abelian Coulomb phase with $3N/2 < N_f < 3N$, which is the conformal window.

3.1 RG flow

The RG equations [22, 11] in the $N \rightarrow \infty$, N/N_f finite limit are determined by the β -functions (where g and λ are 't Hooft couplings)

$$\beta_g = -\frac{g^3}{16\pi^2} \left(3 - \frac{N_f}{N} \right) + \frac{g^5}{(16\pi^2)^2} \left[4 \left(\frac{N_f}{N} \right) - 6 \right] + \dots \quad (3.2)$$

and [14]

$$\beta_\lambda = \lambda[\gamma_\sigma + 2\gamma_Q] \quad (3.3)$$

$$= 2\lambda \left[\left(\frac{N_f}{N} \right) \left(\frac{\lambda}{16\pi^2} \right) - 2 \left(\frac{g^2}{16\pi^2} \right) + \dots \right] \quad (3.4)$$

where γ_σ and γ_Q are the anomalous dimensions of σ and Q respectively. Note, that due to the $\mathcal{N} = 1$ supersymmetry, only the anomalous dimensions determine β_λ , as there is no vertex renormalization. In contrast to (2.4) and (2.5), we see that $a_2 \equiv 0$, which has consequences for the RG flow. Asymptotic freedom of the non-Abelian Coulomb phase requires

$$\frac{3N}{2} < N_f < 3N$$

and

$$0 \leq \lambda < 2 \left(\frac{N}{N_f} \right) g^2. \quad (3.5)$$

A necessary condition for an IR fixed point, with $\lambda \neq 0$, is $\gamma_\sigma + 2\gamma_Q = 0$ at the fixed point.

The solution of the RG equations (3.2) – (3.4) proceeds exactly as described in Sec. 2. There is a non-trivial IR fixed point for g^2 , which for $N_f/N = 3 - \epsilon$, with $\epsilon \ll 1$, is

$$g_*^2 = \frac{8\pi^2}{3} \epsilon + \mathcal{O}(\epsilon^2). \quad (3.6)$$

If $0 < \lambda(M) < 2(N/N_f)g^2(M)$, the RG flow for $g^2(M)$ drives the flow for $\lambda(M)$ as in Sec. 2. Here we have

$$\beta_\lambda = a_0 \lambda^2 - a_1 g^2 \lambda \quad (3.7)$$

with

$$\begin{cases} (4\pi)^2 a_0 = 2 \left(\frac{N_f}{N} \right) + \dots \\ (4\pi^2) a_1 = 4 + \dots \\ (4\pi^2) a_2 \equiv 0 \end{cases}$$

Then defining $y(s) = \lambda(s)/g^2(s)$ as before,

$$\begin{aligned} \frac{dy}{ds} &= a_0 [y - y_+(s)][y - y_-(s)] \\ &= a_0 y [y - y_+(s)] \end{aligned} \quad (3.8)$$

where

$$y_+(s) = \left[\frac{a_1 - b_0 + b_1 x(s)}{a_0} \right]$$

and

$$y_-(s) = 0. \quad (3.9)$$

Thus, the flow is as in Fig. 1, except the lower phase boundary is $y_-(s) = 0$. Hence, here λ may be smoothly turned off, in contrast to the model of Sec. 2. Therefore, we have an IR fixed point g_*^2 and λ_* , which, in analogy with the argument of Seiberg [9], should extend throughout the range $3N/2 < N_f < 3N$.

Given such an IR fixed point, one can use the superconformal algebra to relate exact results for the dimensions D of operators, with the R -symmetry charge R . They satisfy $D \geq \frac{3}{2}|R|$, with $D = \frac{3}{2}R$ for chiral operators. The anomaly free global symmetry is

$$SU(N_f) \times SU(N_f) \times U(1)_B \times U(1)_R ,$$

where the quark chiral superfields transform as²

$$\begin{aligned} Q & : \left(N_f, 1, 1, \frac{N_f - N}{N_f} \right) \\ \tilde{Q} & : \left(1, \bar{N}_f, -1, \frac{N_f - N}{N_f} \right) \end{aligned} \quad (3.10)$$

and

$$\sigma : \left(1, 1, 0, \frac{2N}{N_f} \right) .$$

This implies that the gauge invariant operator $Q\tilde{Q}$ has dimensions

$$D(\tilde{Q}Q) = 3 \left(\frac{N_f - N}{N_f} \right) \quad (3.11a)$$

$$= 2 - \epsilon/3 \quad \text{for } \epsilon \ll 1 . \quad (3.11b)$$

Since W at the fixed point has $R = 2$,

$$D(\sigma) = \frac{3N}{N_f} \quad (3.12a)$$

$$= 1 + \epsilon/3 \quad \text{for } \epsilon \ll 1 . \quad (3.12b)$$

Note that $D(\sigma) > 1$ as is required for an interacting gauge invariant chiral operator. At the other end of the conformal window

$$\begin{cases} D(\tilde{Q}Q) \rightarrow 1 \\ D(\sigma) \rightarrow 2 \end{cases} \quad \text{as } N_f/N \rightarrow 3/2 . \quad (3.13)$$

3.2 Seiberg duality

Section 3.1 describes the theory in electric variables. Following Seiberg [9], one also expects a dual magnetic description of the same theory, even when $\lambda \neq 0$, where again there is an IR fixed point in these variables. The dual group is $SU(\hat{N}) = SU(N_f - N)$, with N_f flavors of quarks q and \tilde{q} , which are not elementary, but non-polynomial functions of Q and \tilde{Q} , with the magnetic description more natural in the range $3N/2 \leq N_f < 2N$. In addition there is a gauge singlet meson M_i^j which

²Although we call the chiral superfields quarks, $Q = \phi + \theta\psi_a + \theta^2 F$, so that the bosonic component behaves as in the vector model.

appears in the dual description, as well as the dual of σ , denoted by $\hat{\sigma}$. The assignments of the quantum numbers of the global symmetry group are

$$\begin{aligned}
q &: \left(\bar{N}_f, 1, \frac{N}{N_f - N}, \frac{N}{N_f} \right) \\
\tilde{q} &: \left(1, N_f, \frac{-N}{N_f - N}, \frac{N}{N_f} \right) \\
M &: \left(N_f, \bar{N}_f, 0, 2 \left(\frac{N_f - N}{N_f} \right) \right) \\
\hat{\sigma} &: \left(1, 1, 0, 2 \left(\frac{N_f - N}{N_f} \right) \right).
\end{aligned} \tag{3.14}$$

The gauge singlets have the superpotentials

$$W = \sqrt{\frac{\hat{\lambda}}{\hat{N}}} \hat{\sigma} \sum_{i=1}^{N_f} \tilde{q}^i q_i$$

and

$$W' = \sqrt{\frac{\lambda'}{\hat{N}}} M_i^i q_i \tilde{q}^i \tag{3.15}$$

where $\hat{N} = (N_f - N)$ and $(\hat{g})^2$, $\hat{\lambda}$ and λ' are the 't Hooft couplings in the magnetic description.

The β -functions of the magnetic description, in the limit $N \rightarrow \infty$ and N_f/N fixed, are

$$\begin{aligned}
\beta_{\hat{g}} &= -\frac{1}{2} \left(\frac{\hat{g}^3}{16\pi^2} \right) \left(\frac{N_f}{N} \right) \\
&+ \frac{2\hat{g}^5}{(16\pi^2)^2} \left(3 - \frac{N_f}{N} \right) \left(1 + \frac{N_f}{N} \right) + \dots
\end{aligned} \tag{3.16}$$

and

$$\beta_{\hat{\lambda}} = 2\hat{\lambda} \left[\left(\frac{N_f}{N_f - N} \right) \frac{\hat{\lambda}^2}{(16\pi^2)} - 2 \left(\frac{\hat{g}^2}{(16\pi^2)} \right) + \dots \right]. \tag{3.17}$$

From an argument due to Leigh and Strassler [13], the flows for $\hat{\lambda}$ and λ' are proportional to each other. Therefore the RG flow is entirely analogous to that described in Sec. 3.1, with asymptotic freedom requiring $\frac{3N}{2} < N_f < 3N$ and $0 < \hat{\lambda} < 2(1 - N/N_f)\hat{g}^2$. The RG flow may again be visualized as in Fig. 1, with the lower phase-boundary $y_-(s) = 0$. If $\frac{N_f}{N} = \frac{3}{2} + \hat{\epsilon}$, with $\hat{\epsilon} \ll 1$, the flow in \hat{g} and $\hat{\lambda}$ is in the domain of perturbation theory. Thus, the addition of (3.1) to the theory does not break Seiberg duality.

4 AdS₅ and gauged vector models

Klebanov and Polyakov (KP) [2] suggested a general relation between theories with an infinite number of higher-spin massless gauge fields in AdS_{*d*+1} and large N conformal theories in d -dimensions containing N -component fields with an infinite number of conserved currents. In particular they

focused on the singlet sector of the 3-d $O(N)$ vector model, and proposed that in the large N limit, the vector model was dual to the minimal bosonic theory in AdS_4 containing massless gauge fields of even spin [2, 15]. This proposal has been generalized by extending the discussion to $\mathcal{N}=1$ supersymmetry [16], as well as to a consistency check of the idea [17].

The study of massless higher-spin theories has evolved over many years, beginning with the work of Fronsdal [5], and of Fradkin and Vasiliev [3]. This program has been generalized in several different ways [18, 19]. The ideas of KP involve the holographic duals of massless higher-spin theories in AdS_4 . Little is known about such theories so far, and their representations are in turn simpler than those in AdS_5 . Therefore our interest in the relationship of gauged vector models in 4-dimensions to higher-spin massless gauge theories is quite speculative. Nevertheless it is interesting to explore these issues, despite our concerns.

Motivated by the work of KP, we wish to find duals to large N conformal theories containing N component fields rather than $N \times N$ matrix fields. As discussed in the introduction, the $O(N)$ invariant vector model in $d = 4$ has many difficulties [6, 10], as well as the absence of an IR fixed point. For these reasons we considered the gauged vector model in Sec. 2, and an $\mathcal{N}=1$ extension thereof in Sec. 3. Since we are not assured that the non-supersymmetric model is conformally invariant at the IR fixed point, we will now confine our discussion to the $\mathcal{N}=1$ example of Sec. 3. If it can eventually be shown that the model in Sec. 2 is conformal at the IR fixed point, then that theory could be analyzed as well by the methods of this section.

The $\mathcal{N}=1$ theory has a spin-zero current $J = Q^i \bar{Q}_i$, which is a flavor and color singlet. [This is called spin-zero as its lowest component is the bosonic spin-zero “current”.] The dimension of the current is

$$D(J) = 3 \left(1 - \frac{N}{N_f} \right) . \quad (4.1)$$

Further in the large N limit, the anomalous dimension $D(J)$ is known in perturbation theory, *i.e.*,

$$D(J) = 2 - \frac{g^2}{8\pi^2} + \mathcal{O}(g^4) . \quad (4.2)$$

At the IR fixed point, for $\epsilon \ll 1$, one has

$$\begin{aligned} D_{IR}(J) &= 2 - \frac{g_*^2}{8\pi^2} + \mathcal{O}(g_*^4) \\ &= 2 - \frac{\epsilon}{3} + \mathcal{O}(\epsilon^2) \end{aligned} \quad (4.3)$$

while near the UV fixed point

$$D_{UV}(J) = 2 - \frac{g^2}{8\pi^2} + \mathcal{O}(g^4) . \quad (4.4)$$

Following KP, we conjecture that the correlation functions of the singlet currents at the conformal UV or IR fixed points may be obtained from AdS_5 from an AdS/CFT prescription. The dimensions of scalar fields in AdS_5 is

$$\Delta_{\pm} = 2 \pm \sqrt{4 + (mL)^2} \quad (4.5)$$

where L is the radius of AdS_5 . Near the UV(IR) fixed point we identify

$$D_{UV}(J) = (\Delta_-)_{UV}$$

and

$$D_{IR}(J) = (\Delta_-)_{IR} \quad (4.6)$$

which implies, from (4.3)–(4.5), that

$$(mL^2)_{UV} = -4 + (g^2/8\pi^2)^2 + \dots$$

and

$$(mL^2)_{IR} = -4 + (g_*^2/8\pi^2)^2 + \dots \quad (4.7)$$

for $\epsilon \ll 1$. Further, from the RG flow, one has $0 \leq g^2 \leq g_*^2$, so that

$$0 < |m^2 L^2|_{IR} < |m^2 L^2|_{UV} , \quad (4.8)$$

which is consistent with the RG flow in the bulk. Suppose that the current J is dual to a scalar field h in AdS_5 [1], with action

$$S(h) = \frac{N}{2} \int d^5x \sqrt{g} [(\partial_\mu h)^2 + m^2 h^2 + \dots] \quad (4.9)$$

where $m^2 = -4/L^2 + \mathcal{O}(g^4)$, and L is the AdS_5 radius. This identification required us to choose the Δ_- in equations (4.5)–(4.8) for the CFT, since the anomalous dimension of the operator $D(J)$ is negative. [We do not have an interpretation of Δ_+ in the CFT analogous to that of KP [2].] There is another independent singlet scalar current present if σ is a dynamical chiral superfield. Define

$$\Sigma = (\sigma\sigma) \quad (4.10)$$

whose dimension at the IR fixed point can be expressed in terms of Δ_+ , (4.5), but with a perturbative correction. At the IR fixed point of β_λ , (3.3) implies that the anomalous dimensions satisfy

$$\gamma_\sigma + \gamma_J = 0 . \quad (4.11)$$

This means that

$$\begin{aligned} D_{IR}(\Sigma) &= 2 D_{IR}(\sigma) = 2 - \gamma_J + \gamma_\sigma \\ &= 2 + \left(\frac{g_*^2}{8\pi^2} \right) + \left(\frac{N_f}{N} \right) \left(\frac{\lambda_*}{16\pi^2} \right) + \dots \end{aligned} \quad (4.12)$$

Then (4.3) and (4.5) give

$$\begin{aligned} (\Delta_+)_{IR} &= 2 + \sqrt{4 + (mL)^2} \\ &= 2 + \left(\frac{g_*^2}{8\pi^2} \right) + \dots \end{aligned} \quad (4.13)$$

Hence

$$\begin{aligned} D_{IR}(\Sigma) &= (\Delta_+)_{IR} + \gamma_\sigma \\ &= (\Delta_+)_{IR} + \left(\frac{N_f}{N} \right) \left(\frac{\lambda_*}{16\pi^2} \right) + \dots \end{aligned} \quad (4.14)$$

Similarly, near the UV fixed point

$$D_{UV}(\Sigma) = (\Delta_+)_{UV} + \left(\frac{N_f}{N}\right) \left(\frac{\lambda}{16\pi^2}\right) + \dots \quad (4.15)$$

If one associates Σ to a dual field H in AdS_5 with mass μ , satisfying

$$D(\Sigma) = 2 + \sqrt{4 + (\mu L)^2} \quad (4.16)$$

then

$$\mu^2 = -4/L^2 + \mathcal{O}(g^4, g^2\lambda, \lambda^2) \quad (4.17)$$

and so

$$\mu^2 = m^2 + \mathcal{O}(g^4, g^2\lambda, \lambda^2) . \quad (4.18)$$

As in (4.8), we also have

$$0 < |\mu^2 L^2|_{IR} < |\mu^2 L^2|_{UV} .$$

consistent with RG flow in the bulk. The total action for the scalar fields h and H in AdS_5 is then conjectured to be

$$S = \frac{N}{2} \int d^5x \sqrt{g} [(\partial_\mu h)^2 + (\partial_\mu H)^2 + m^2 h^2 + \mu^2 H^2 + \dots] \quad (4.19)$$

where the interaction terms should include mixing between h and H , as well as self-interactions.

The discussion of this section has focused on the electric description of the theory. For $N_f/N = 3/2 + \hat{\epsilon}$, with $\hat{\epsilon} \ll 1$, the magnetic description should be more useful. Define the scalar superfields for the magnetic description,

$$\tilde{J} = q^i \tilde{q}_i$$

and

$$\tilde{\Sigma} = \hat{\sigma} \hat{\sigma} . \quad (4.20)$$

Then

$$\begin{aligned} D(\tilde{J}) &= 3N/N_f \\ &= 2 - \frac{4}{3} \hat{\epsilon} + \mathcal{O}(\hat{\epsilon}^2) \end{aligned} \quad (4.21)$$

and

$$\begin{aligned} D(\tilde{\Sigma}) &= 3(1 - N/N_f) \\ &= 1 + \frac{4}{3} \hat{\epsilon} + \mathcal{O}(\hat{\epsilon}^2) . \end{aligned} \quad (4.22)$$

In a discussion entirely parallel to the electric description, one associates the scalar fields \tilde{h} and \tilde{H} in AdS_5 to \tilde{J} and $\tilde{\Sigma}$, with an action of the form of (4.19), involving \tilde{h} and \tilde{H} , and masses \tilde{m} and $\tilde{\mu}$, where

$$\begin{aligned} \tilde{m}^2 &= -4/L^2 + \mathcal{O}(\hat{g}^4) \\ \tilde{m}^2 &= \tilde{\mu}^2 + \mathcal{O}(\hat{g}^4, \hat{g}^2 \hat{\lambda}, \hat{\lambda}^2) . \end{aligned} \quad (4.23)$$

If N_f/N is not at either end of the conformal window $3/2N < N_f < 3N$, then a perturbative expansion in terms of (g^2, λ) or $(\hat{g}^2, \hat{\lambda})$ is not likely to be rapidly converging, so that the description in terms of scalar fields in AdS_5 is likely to involve the omitted interactions in (4.19) in an essential way. Therefore, it may be difficult to give a suitable explicit completion of (4.19) valid outside the perturbative end-points of the conformal window.

5 Infinite Spin Representations

The theories described in Secs. 2 and 3 have a class of $U(N)$ or $SU(N)$ respectively, gauge and flavor singlet conserved currents at the UV fixed point ($\hat{g} = \lambda = 0$),

$$J_{(\mu_1 \dots \mu_s)} = \phi^a \overleftrightarrow{D}_{(\mu_1 \dots \mu_s)} \phi_a + \text{fermion terms} \quad (5.1)$$

for each spin s , where D_μ is the gauge covariant derivative. The model of Sec. 3 has an additional class of conserved currents of even spins only,

$$\Sigma_{(\mu_1 \dots \mu_s)} = \chi \partial_{(\mu_1 \dots \mu_s)} \chi + \text{fermion terms} \quad (5.2)$$

where in (5.2) χ is the bosonic component of the chiral superfield σ . Following closely KP [2], and earlier workers [18], one conjectures that the correlation functions of these singlet currents in the free 4-d theories can be obtained from a bulk action in AdS_5 , through the AdS/CFT property with relates the boundary values of fields with that of sources $h^{(\mu_1 \dots \mu_s)}$ in the dual field theory. That is

$$\left\langle \exp \int d^4x h_0^{(\mu_1 \dots \mu_s)} J_{(\mu_1 \dots \mu_s)} \right\rangle = e^{S[h_0]} \quad (5.3)$$

where $S[h_0]$ is the action of a high-spin gauge theory in AdS_5 , given in terms of the boundary values h_0 of fields. [For spin zero, this action is given by (4.19).]

In order to understand the action for the bulk fields $h_{(\mu_1 \dots \mu_s)}$ one needs an appropriate representation theory for an infinite tower of higher-spin massless-representations in AdS_5 . For our purposes, the work of Sezgin and Sundell (SS) [4] seems most suitable. Representations of $\mathcal{N}=1$ supermultiplets in AdS_5 are classified by $SU(2,2|1)$, while the truncation to the bosonic components of the currents (5.1) and (5.2) transform according to $SO(4,2)$, whose group theory has been analyzed by SS [4].

Denote the bosonic components of the chiral superfields Q and σ of Sec. 3 as ϕ and χ respectively. At the UV fixed point, *i.e.*, free field limit, the dimensions of ϕ and χ are

$$\begin{aligned} \Delta(\phi) &= \Delta(\chi) = 1 \\ &= 1 + j \quad \text{with } j = 0. \end{aligned} \quad (5.4)$$

Thus ϕ and χ are a pair of doubleton representations of $SO(4,2)$ [4, 5], denoted by $D_\phi(0,0;1)$ and $D_\chi(0,0;1)$ respectively. As SS [4] show, one obtains states with even spins $s = j_L + j_R = 0, 2, 4, \dots$ from the symmetric tensor product of spin-zero doubleton representations, and odd spins from the anti-symmetric product. The maximal compact subgroup of $SO(4,2)$ is $SU(2)_L \times SU(2)_R \times U(1)_R$ whose weight spaces are labeled $D(j_L, j_R; \Delta)$, with lowest weight states $|j_L, j_R; \Delta\rangle$, where $\Delta = \frac{3}{2}R$ for chiral operators. Then

$$\begin{aligned} [D(0,0;1) \otimes D(0,0;1)]_S \\ = \sum_{s \text{ even}} D\left(\frac{s}{2}, \frac{s}{2}; s+2\right) \end{aligned} \quad (5.5)$$

$$\begin{aligned} [D(0,0;1) \otimes D(0,0;1)]_A \\ = \sum_{s \text{ odd}} D\left(\frac{s}{2}, \frac{s}{2}; s+2\right). \end{aligned} \quad (5.6)$$

We conjecture that the bosonic components of the currents (5.1) and (5.2) are classified by

$$D_\phi(0, 0; 1) \otimes D_\chi(0, 0; 1) \quad (5.7)$$

and

$$[D_\chi(0, 0; 1) \otimes D_\chi(0, 0; 1)]_S \quad (5.8)$$

respectively, with the gauge and flavor singlets being selected in (5.7). Since χ is real, we only keep the symmetric product of (5.6) in constructing (5.8), while (5.7) requires both (5.5) and (5.6). Notice that $s = 0$, $\Delta = 2$ in (5.5) which implies for the bulk states $m^2 = \mu^2 = -4/L^2$, corresponding to the spin-zero currents J and Σ of Sec. 3.

Sezgin and Sundell [4] build their representations from the four-component $\text{SO}(4,1)$ Dirac spinor y_α , and its conjugate \bar{y}^α . They then extend this representation to $\text{SU}(2,2|1)$ by introducing an additional set of Grassman odd complex oscillators θ , forming a Clifford algebra and an odd supercharge $Q_\alpha = y_\alpha \theta$. This generates the supersymmetric extension of $\text{hs}(2,2)$ to $\text{hs}(2,2|1)$ by means of the $\mathcal{N}=1$ version of the product (5.5) and (5.6). We do not present the details here.

The above discussion is applicable to the UV fixed point, where $g = \lambda = 0$. Now consider the gauge singlet currents (5.1) and (5.2) for $g^2 \neq 0$, $\lambda \neq 0$ (not at either the UV or IR fixed point). The gauge and flavor singlet bosonic current $J_{(\mu\nu)}$ has a dependence on the gauge field given by

$$\begin{aligned} J_{(\mu\nu)} &= \phi \overleftrightarrow{D}_{(\mu} \overleftrightarrow{D}_{\nu)} \phi \\ &= \phi \overleftrightarrow{\partial}_{(\mu} \overleftrightarrow{\partial}_{\nu)} \phi + g^2 \phi A_\mu^\alpha A_\nu^\beta \{T_\alpha, T_\beta\} \phi \\ &= \phi \overleftrightarrow{\partial}_{(\mu} \overleftrightarrow{\partial}_{\nu)} \phi + \frac{g^2}{N} \phi A_\mu^\alpha A_\nu^\alpha \phi + g^2 d_{\alpha\beta\gamma} A_\mu^\alpha A_\nu^\beta (\phi T_\gamma \phi) \end{aligned} \quad (5.9)$$

where g^2 is the 't Hooft coupling. In (5.9) T_α is a generator of the gauge group $\text{SU}(N)$ in the fundamental representation, satisfying

$$[T_\alpha, T_\beta] = i f_{\alpha\beta\gamma} T_\gamma$$

and

$$\{T_\alpha, T_\beta\} = \frac{1}{N} \delta_{\alpha\beta} + d_{\alpha\beta\gamma} T_\gamma. \quad (5.10)$$

This can be easily generalized to all of the currents of (5.1) where the current (5.9) and its generalization to higher spins is defined with normal ordered fields. Therefore, the gauge field contribution to the singlet part of the current is suppressed by a factor of $1/N$. [Recall the rescaling of fields in (2.1), and in Sec. 3.] Thus, neglect of the gauge fields in the gauge singlet, flavor singlet part of the currents appears to be a consistent truncation, correct to leading order in $1/N$.

There are a very large number of other operators in the CFT involving gauge field strengths, which are gauge and flavor singlets. Among these are

$$\text{tr} F_{(\mu_1}^\alpha \overleftrightarrow{D}_{\mu_2} \dots \overleftrightarrow{D}_{\mu_{s-1}} F_{\mu_s)}^\alpha \quad (5.11)$$

which have dimensions $\Delta = 2 + s$ ($\Delta \geq 4$). Another example is

$$(F^2) = F_{\mu\nu}^{\alpha\beta} F_{\mu\nu}^{\alpha\beta} \quad (5.12)$$

which is dual to the dilation, which has $\Delta=4$ and AdS_5 mass $M^2 = 0$. There is also a double-trace operator

$$(F^2 J) = F_{\mu\nu}^{\alpha\beta} F_{\mu\nu}^{\alpha\beta} J + \text{fermi terms}, \quad (5.13)$$

with dimensions $\Delta(F^2 J) = 6$ at the UV fixed point, corresponding to $M^2 = 4/L^2$. There are infinite number which generalize (5.12) or (5.13) involving additional field strengths. Other operators are typified by

$$d_{\alpha\beta\gamma} F_{\mu\nu}^{\alpha\beta} F_{\mu\nu}^{\beta\alpha} (\phi T_\gamma \phi) + \text{fermi terms} , \quad (5.14)$$

with $D=6$, but are not gauge singlets in the matter fields, so are omitted in our truncation to gauge, flavor singlets. Similar remarks apply to the higher-spin fields analogous to (5.1), *i.e.*, operators such as $\text{tr}(F^2) J_{(\mu_1 \dots \mu_s)}$, etc. Operators (5.11)–(5.14) and their generalizations cannot be represented by (5.5) or (5.6). In order to include them one will need to consider appropriate generalizations of the representation theory. This is work for the future.

If one is not at the UV fixed point, the currents (5.1) and (5.2) are not conserved. Rather their divergence can be expressed as a power series in g^2 and λ , with λ related to g^2 on the RG flow. This suggest a Higgs-like mechanism for the bulk gauge fields $h_{(\mu_1 \dots \mu_s)}$. For even spins, the currents (5.1) and (5.2) will mix, as will their bulk gauge field duals. The details of the relevant Higgs mechanism remains to be worked out, and is an unsolved problem.

At the IR fixed point the theory is superconformal. A generic superconformal primary satisfies unitarity bounds given in [20]. The unitarity thresholds are satisfied by massless fields, and conserved tensor fields. However, due to the anomalous dimensions at the IR fixed point, this is not the case for the currents (5.1) or (5.2). Since the $\mathcal{N} = 1$ currents (5.1) and (5.2) have anomalous dimensions at the IR fixed point, they are not conserved, and are no longer the products of fields which saturate the unitarity lower-bound. This then opens into question the group theory of (5.5) and (5.8), as the currents are no longer the products of doubleton representations at the IR fixed point. Perhaps a straightforward generalization of (5.5)–(5.8) for currents with anomalous dimensions is possible. This is a project for the future.

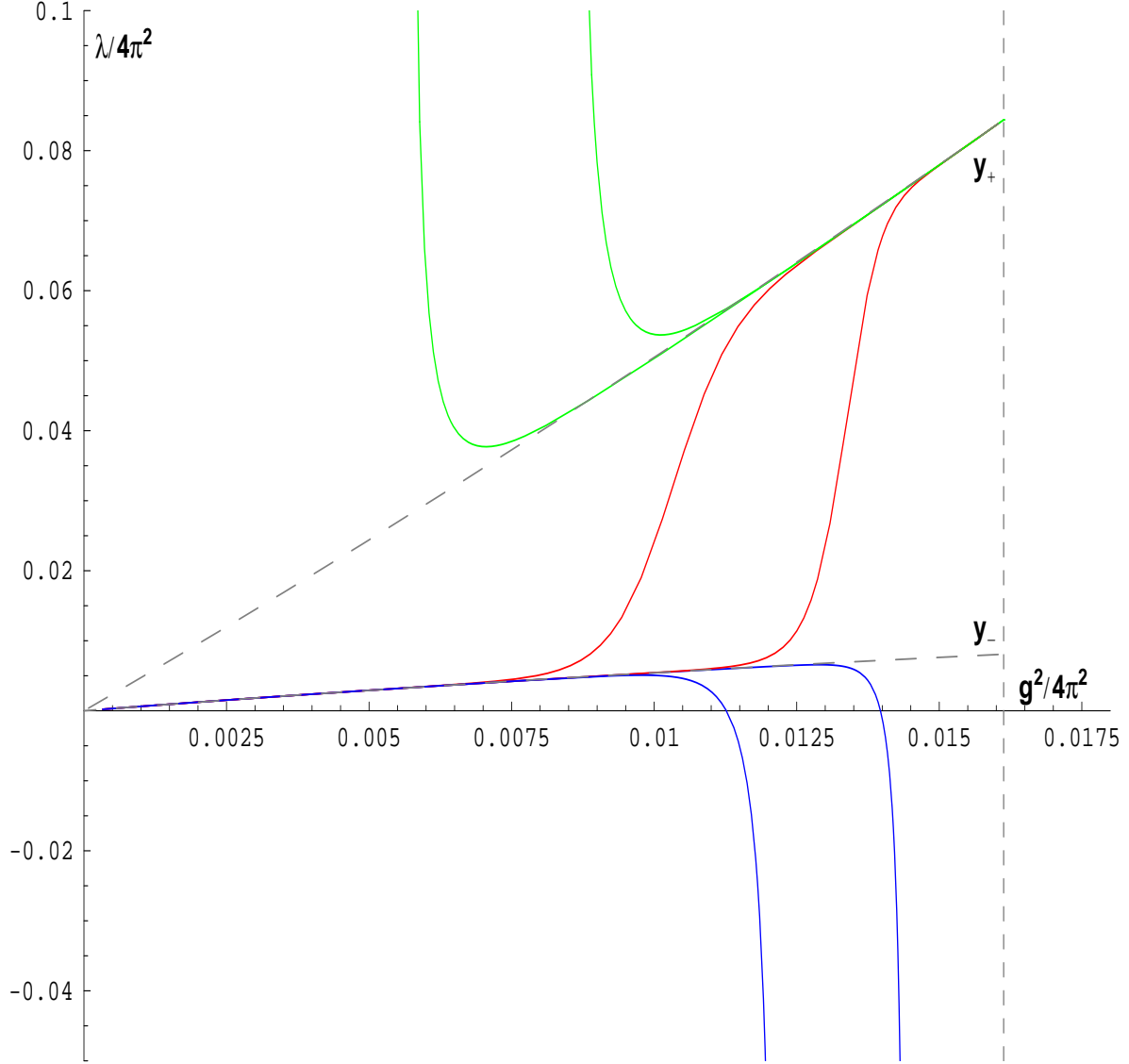
6 Concluding Remarks

We have presented the large N gauged vector model in four-dimensions, and an $\mathcal{N}=1$ supersymmetric analogue, and studied the RG flow between the UV and IR fixed points of the theories. It was speculated that AdS_5 duals to the gauge and flavor singlet currents of the model at the UV fixed point could be characterized by the higher-spin representations of a type proposed by Vasiliev and others [3], and studied in detail for AdS_5 by Sezgin and Sundell [4]. The higher-spin currents at the UV fixed-point are conjectured to be dual to direct products of doubleton representations of $\text{SU}(2,2|1)$. The infinite spin gauge invariance is broken as one moves away from the UV fixed point. The higher-spin currents are no longer conserved, but are possibly Higgsed, with their divergences proportional to a power series in the 't Hooft couplings, as appropriate to a small radius expansion of AdS_5 ($1/\alpha' \ll 1$). At the IR fixed point the theory is conformal, though the currents have anomalous dimensions. Therefore the representation theory will require a generalization of the work of SS [4], which is not available as yet.

The discussion of Secs. 4 and 5 is speculative, and requires more work to clarify the status of those ideas. Further, we do not have an understanding from the point of view of dual bulk representations of the non-singlet currents of the model. Clearly additional attention to these issues is required, even if the overall picture presented here is correct.

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Graph of the renormalization group flow for $N_f/N = 5$, where ultraviolet to infrared flow progressing from left to right. The upper and lower dashed lines, y_+ and y_- respectively, brackets flows consistent with asymptotic freedom and stability of the theory. The vertical dashed line at the right marks the value of the infrared fixed-point g_* .

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